Active suppression of dephasing in Josephson-junction qubits

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Simple majority code correcting k dephasing errors by encoding a qubit of information into $2k + 1$ physical qubits is studied quantitatively. We derive an equation for quasicontinuous evolution of the density matrix of encoded quantum information under the error correction procedure in the presence of dephasing noise that in general can be correlated at different qubits. Specific design of the Josephson-junction circuit implementing this scheme is suggested.

Josephson qubits are among the most promising devices to implement solid state quantum computation [1,2]. Quantum manipulations of individual [3,4,5,6,7,8,9] and coupled [10] qubits has been demonstrated experimentally. At present, probably the main obstacle to the development of a larger-scale solid state quantum logic circuits is presented by decoherence. It is therefore important to develop strategies to minimize the effects of decoherence on the dynamics of the qubit systems.

Known approaches to reduction of decoherence include both error-correction and error-avoiding schemes that either employ symmetries of the qubit-environment interaction to create areas of the Hilbert space not affected by decoherence [11,12] or use rapid random dynamic perturbations of the system to average out the effects of external noise [13,14]. The error-avoiding approaches appear to be less suitable for the solid-state qubits. Indeed, noise in solid-state systems typically does not have any particular symmetry and its correlation time is short, so that the application of the control pulses within this time-scale, as required by the dynamic averaging schemes, is problematic. This leaves error-correction as the main strategy for suppression of decoherence in solid-state qubits. In this work, we suggest an implementation of one of the basic error-correction algorithms for the suppression of dephasing errors (which can be expected to be the dominant type of errors in solid-state circuits - see, e.g., Ref. [9]), and develop its quantitative description. Our scheme employs the Josephson-junction qubits that combine charge and flux dynamics [15,16,6], and requires only a small number of qubit transformations to operate.

From the perspective of the general theory of error-correction, an interesting feature of the scheme considered in this work is the possibility of developing its detailed quantitative description within the realistic model of the qubit-environment interaction and analyzing, for instance, the effect of the correlations in the noise acting on different qubits. While discussions of the error-correction rely typically in an essential way on independent noise models, environments of the solid-state qubits can be to a large degree correlated because of the finite distance between the qubits in a circuit. A clear illustration of this is provided by the background charge fluctuations that are the main source of dephasing in charge qubits [3,6,10,17]. Long-range nature of the Coulomb interaction creates noise correlations by coupling the qubits to the same charge fluctuators.

We consider specifically the problem of "quantum memory", when the task is to preserve the stationary state of the qubit in the presence of dephasing noise. The qubit Hamiltonian contains then only the coupling to the environment. Under the assumption that the environment has many degrees of freedom each of which is only weakly coupled to qubits, it can be modeled as an ensemble of harmonic oscillators [18,19,20,21] (see however [17]), so that the Hamiltonian of the qubit register is:

$$H = \sum_j \sigma_z^{(j)} \xi_j \,, \qquad (1)$$

where $\xi_j = \sum_{m,k}[\lambda_{m,j}(\omega)a_{m,\omega} + \text{h.c.}]$. Here we assumed several independent ensembles of environmental oscillators (numbered by m), as needed to model different profiles of spatial correlations of random forces ξ_j. The index $j = 1, 2...$ in (1) numbers the qubits, and coefficients $\lambda_{m,j}(\omega)$ are coupling constants of the qubit j to the oscillators of reservoir m in the mode ω and creation/annihilation operators $a_{m,\omega}, a_{m,\omega}^\dagger$. Time evolution of the "qubits+environment" system is described conveniently in the interaction representation with respect to the interaction Hamiltonian of Eq. (1). The evolution operator $U(t)$ can then be calculated explicitly by separating the two non-commuting parts, $a_{m,\omega}$, and $a_{m,\omega}^\dagger$, of the qubit-oscillator coupling, and using the fact that their commutator is a c-number:

$$U(t) = \exp\{-i \sum_j \varphi_j(t)\sigma_z^{(j)}\}U_r(t) \,, \qquad (2)$$

$$U_r(t) = \exp\{i \sum_{m,\omega} \frac{\omega t - \sin \omega t}{\omega^2} |\sum_j \lambda_{m,j}\sigma_z^{(j)}|^2\} \,.$$

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The first term in $U(t)$ represents fluctuating phases $\varphi_j(t)$ of the qubit basis states induced by the environmental forces $\xi_j(t)$: $\varphi_j(t) = \int_0^t \xi_j(t')dt'$. The second term, $U_r(t)$, results from the renormalization of the qubit parameters by the qubit-environment interaction. To see this more explicitly, we note that the sum over frequencies ω in this exponent has a natural cut-off at large frequencies $\omega \simeq \tau_c^{-1}$, where τ_c is the time scale at which environment forces acting on different qubits are correlated. For weak decoherence we are interested in the time scales much larger that τ_c. In this regime, the phase represented by $U_r(t)$ is dominated by the term that grows linearly with t, and can be viewed as arising from the renormalization of the qubit energy. Equation (2) shows that such a renormalization includes then the shift of the total energy of the register and creation of the qubit-qubit interaction. The total energy shift is irrelevant as long as we consider an individual register. Neglecting it, we see that $U_r(t)$ results from the Hamiltonian evolution with the Hamiltonian

$$H_r = -\sum_{j,j'} V_{jj'} \sigma_z^{(j)} \sigma_z^{(j')} , \qquad (3)$$

and $V_{jj'} = 2\mathrm{Re}\sum_{m,\omega}(\lambda_{m,j}(\omega)\lambda_{m,j'}^*(\omega)/\omega)$, if the sum over frequencies ω in this expression is converging at low frequencies. The qubit-qubit interaction strength $V_{jj'}$ is non-vanishing only if the same reservoir m couples to more than one qubit, so that the reservoir forces ξ_j at different qubits are correlated.

The time evolution with the Hamiltonian H_r, and more generally, the evolution operator U_r in Eq. (2) represent deterministic part of the qubit evolution induced by the qubit-reservoir interaction. As a result, it can in principle be compensated for by adjusting the regular (non-dissipative) part of the Hamiltonian of the qubit register. This procedure, however, is impractical even in the case of constant H_r, since the interaction constants $V_{jj'}$ are apriori unknown and incommesurate quantities. This complexity means that a more appropriate approach is to treat the time evolution represented by U_r as dephasing despite its deterministic character.

The time evolution of the density matrix $\rho(t)$ of the qubit register is obtained from Eq. (2) through the relation $\rho(t) = \mathrm{Tr}_{env}\{U^\dagger(t)\sigma(0)U(t)\}$, where σ is the total density matrix of the "qubits+environment" system. The environment will dephase the qubits if they are prepared initially in the state $\rho(0)$ that is uncorrelated with the state of the environment, $\sigma(0) = \rho_{env}\rho(0)$. Assuming that the environment is in thermal equilibrium at temperature Θ, and no error correction procedure is applied, we get using the standard property of the Gaussian noise:

$$\rho(t) = \exp\{-\frac{1}{2}\sum_{j,j'}\langle\varphi_j(t)\varphi_{j'}(t)\rangle(\sigma_z^{(j)}-\bar\sigma_z^{(j)})(\sigma_z^{(j')}-\bar\sigma_z^{(j')})\}$$
$$\cdot U_r^\dagger(t)\rho(0)U_r(t) . \qquad (4)$$

Here we introduced the convention that the bar over σ_z operators means that they act on ρ from the right. Qualitatively, Eq. (4) shows that the matrix elements of ρ that are further away from the diagonal in the σ_z basis decay faster. The diagonal elements (on which $\sigma_z - \bar\sigma_z = 0$) remain constant. In the case of one physical qubit, Eq. (4) gives $\rho(t) = e^{-\langle\varphi^2(t)\rangle(1-\sigma_z\bar\sigma_z)}\rho(0)$, i.e., the off-diagonal elements of ρ are suppressed with time as $e^{-2\langle\varphi^2(t)\rangle} \equiv e^{-P(t)}$. If the environment density of states is Ohmic, i.e., $\sum_{m,\omega}|\lambda_m(\omega)|^2... = g\int_0^\infty d\omega\omega e^{-\omega\tau_0}...$, direct evaluation for $\Theta \ll 1/\tau_0$ gives: $P(t) = 2g\ln[\sinh(\pi t\Theta)/(\pi\tau_0\Theta)]$. At large t, when the random force ξ appears δ-correlated, $P(t)$ reduces to $P(t) = \Gamma t$, where $\Gamma = 2\pi g\Theta$ is the dephasing rate.

One can reduce the effective dephasing rate by the encoding that corrects the phase errors [22,23]. Generalized to k errors, this encoding is:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|\underbrace{++...+}_{2k+1}\rangle + \beta|\underbrace{--...-}_{2k+1}\rangle . \qquad (5)$$

In Equation (5), a bit of quantum information is encoded in the state of the $2k + 1$ physical qubits, and the $|\pm\rangle$ states of each of these qubits are obtained through the Hadamard transform $\hat{H}$ (the $\pi/2$-rotation around y axis) from the $|0,1\rangle$ states. All of the σ_z operators in the dephasing-induced time evolution (2) are changed by $\hat{H}$: $\hat{H}\sigma_z\hat{H} = \sigma_x$, so that for the states on the right-hand-side of Eq. (5) the dephasing looks like transitions between the $|\pm\rangle$ states of each qubit, and can be directly detected by measurements in this basis and corrected by applying simple pulses returning the qubit into the initial state. The error-detecting measurements, however, should not destroy the quantum information encoded in the state (5), i.e., they should not distinguish the α and β parts of this state. This condition is not satisfied by measurements on individual qubits but can be satisfied by the measurements on pairs of the nearest-neighbor qubits comparing their states. Despite the apparent complexity of this scheme, it has quite natural implementation in the Josephson-junction qubits - see Fig. 1.

To describe this process quantitatively we assume that its measurement/correction part can be done on the time scale that is much shorter than the one set by the characteristic dephasing rate Γ. Different terms in the environment-induced evolution of the encoded state, Eq. (5), during the time interval T between the successive application of the "measurement+correction" operations can be conveniently classified by the number of qubits flipped during this time interval. In the relevant regime of sufficiently short T: $P(T) \ll 1$, the probability amplitude of these terms decreases rapidly when this number increases. If we keep only the terms that flip up to k qubits, we see directly from Eq. (2) that the time evolution at this level of accuracy (denoted by $U_k(T)$) preserves the superposition of the α and β parts of the

encoded state:

$$U_k(T)[\alpha|\oplus\rangle + \beta|\ominus\rangle] = \sum_q [\alpha|\psi_q\rangle + \beta\hat{R}|\psi_q\rangle]u_q. \quad (6)$$

Here index q runs over 2^{2k} different register states obtained from the state $|\oplus\rangle \equiv |+...+\rangle$ by flipping up to k qubits, u_q are the probability amplitudes of these states, $\hat{R}|\psi_q\rangle$ denotes the state $|\psi_q\rangle$ with all $2k+1$ qubits inverted, and $|\ominus\rangle \equiv |-...-\rangle$.

The measurements that compare the qubit states in all pairs of the nearest-neighbor qubits do not distinguish states $|\psi_q\rangle$ and $\hat{R}|\psi_q\rangle$, and therefore also preserve the superposition of the α and β terms in Eq. (6). The 2^{2k} different outcomes ("equal" or "different") of the $2k$ such measurements distinguish all terms with different q in Eq. (6) and enable one to decide what qubits were flipped during the time interval T. Application of the correcting pulses should then bring the state of the qubit register back to its initial form (5) so that the encoded quantum state does not change in this approximation. The residual evolution of the encoded state is associated with the possibility that environment flips more that k different qubits; for $P(T) \ll 1$ – precisely $k+1$ qubits. Following the same steps as above, we see that when $k+1$ qubits are flipped, the measurement/correction cycle interchanges the α and β weights in the encoded state (5). Since the probability p of this mistake is small, $p \ll 1$, the encoded state changes substantially only on the time scale larger than the period T of one error-correction cycle, and its evolution on this scale can be conveniently described by the continuous equation for the density matrix $\rho^{(c)}$ in the basis of $|\oplus\rangle$ and $|\ominus\rangle$ states. The interchange of the α and β terms in (5) leads to the following evolution of $\rho^{(c)}$:

$$\dot{\rho}^{(c)}_{++} = \frac{\gamma_k}{2}(\rho^{(c)}_{--} - \rho^{(c)}_{++}), \quad \dot{\rho}^{(c)}_{+-} = \frac{\gamma_k}{2}(\rho^{(c)}_{-+} - \rho^{(c)}_{+-}). \quad (7)$$

Here $\gamma_k \equiv 2p/T$ is the effective dephasing rate of the encoded quantum information, and the superscript (c) indicates that $\rho^{(c)}$ is the reduced density matrix in the presence of error correction. Thus, our error-correcting procedure replaces the dephasing in the individual physical qubit with the dephasing of encoded quantum information at a smaller rate. Indeed, if one writes Eqs. (7) in the rotated basis $|\oplus\rangle \pm |\ominus\rangle$, they explicitly acquire the form characteristic for pure dephasing: constant diagonal elements of the density matrix and decay of the off-diagonal elements with the rate γ_k. The dephasing rate γ_k can be calculated from the evolution operator (2). Now we discuss several important limits.

For $k = 1$, when the relevant errors flip 2 out of 3 qubits, we get:

$$\gamma_1 = \frac{2}{T}\sum_{j>j'}(T^2V_{jj'}^2 + 2\langle\varphi_j(T)\varphi_{j'}(T)\rangle^2 + \langle\varphi_j^2(T)\rangle\langle\varphi_{j'}^2(T)\rangle).$$

The first two terms in this expression represent contribution to dephasing from noise correlations at different

qubits, while the last term exists also for uncorrelated noise. If the noise is δ-correlated in time, γ_1 reduces to $\gamma_1 = T\sum_{j>j'}(2V_{jj'}^2 + \Gamma_{jj'}^2 + \Gamma_j\Gamma_{j'}/2)$, where Γ_j is the dephasing rate in the jth qubit, and $\Gamma_{jj'}$ is introduced through $2\langle\varphi_j(t)\varphi_{j'}(t)\rangle = \Gamma_{jj'}t$.

For $k = 2$, the dephasing rate of the encoded state is:

$$\gamma_2 = \frac{2}{T}\sum_{j>j'>j''}\langle\varphi_j^2\varphi_{j'}^2\varphi_{j''}^2\rangle. \quad (8)$$

Since the effective coupling induced by the environment – see Eq. (3), flips the qubits only in pairs, it does not contribute to γ_2. If the dephasing noise is δ-correlated in time, its space correlations are non-vanishing only for the nearest-neighbor qubits, and the corresponding dephasing rates are the same for all qubits, Eq. (8) gives: $\gamma_2 = 5\Gamma T^2(\bar{\Gamma}^2 + \Gamma^2/2)$, where $\bar{\Gamma} \equiv \Gamma_{jj+1}$.

If the dephasing forces at different qubits are uncorrelated, the encoded dephasing rate can be easily calculated for arbitrary k:

$$\gamma_k = \frac{1}{2^kT}\sum_{j_1>j_2>...>j_{k+1}}P_{j_1}(T)P_{j_2}(T)...P_{j_{k+1}}(T), \quad (9)$$

and one sees that γ_k decreases exponentially with the "degree of encoding" k. When the probabilities $P(T)$ of dephasing errors in individual qubits can be expressed through the dephasing rate Γ, Eq. (9) reduces to $\gamma_k = \Gamma(\Gamma T)^k(2k+1)!/(2^k k!(k+1)!)$, if Γ is the same for all qubits.

Exponential suppression of γ_k with k is limited in the scheme considered above by possible imperfections of the measurement/correction operations. The most important is direct dephasing of the encoded state by measurements, which, in contrast to correction steps, need to be performed each period T. For example, one of the specific non-idealities of measurement detectors that leads to direct dephasing of the encoded state is residual linear response coefficient of the quadratic detectors needed to perform pair-wise comparison of the qubit states – see Eq. (11) below. Linear terms couple the detector directly to the $|\pm\rangle$ states of individual qubits and introduce finite phase shifts between them. Since the number of required measurements is proportional to k, the rate of introduced dephasing should also be proportional to k, and can be denoted as $\bar{\gamma}k$. The effect of this dephasing on the evolution of the encoded quantum information is described then by adding the usual dephasing term to the equation for the off-diagonal element of the density matrix $\rho^{(c)}$ (7):

$$\dot{\rho}^{(c)}_{+-} = \frac{\gamma_k}{2}(\rho^{(c)}_{-+} - \rho^{(c)}_{+-}) - \bar{\gamma}k\rho^{(c)}_{+-}. \quad (10)$$

Qualitatively, the two types of dephasing processes in Eq. (10) have similar effect of suppressing the fidelity of the encoded state, but depend differently on k. The optimum degree of encoding is estimated crudely by minimizing

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the total dephasing rate: $k_{opt} \sim \ln(\bar{\gamma}/\Gamma)/\ln(T\Gamma)$. One obvious result of this optimization is that for the considered scheme of the dephasing suppression to make sense, the dephasing introduced by imperfections of the correcting procedure should be much weaker than the original qubit dephasing Γ.
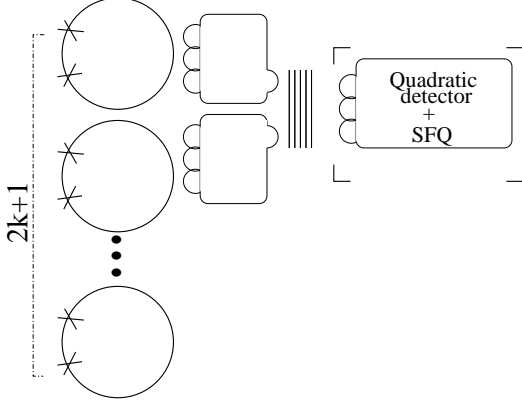

FIG. 1. Schematics of the Josephson-junction circuits implementing suppression of dephasing. Crosses denote tunnel junctions, the electrodes between them act as charge qubits. Monitored currents in the nearest-neighbor loops enclosing qubits allow to detect dephasing errors.

This condition can be satisfied in Josephson-junction qubits, where the dynamics of magnetic flux characterized by longer coherence times (at least tens of nanoseconds – see, e.g., [9]) can be used to suppress dephasing in charge-based qubits. The charge qubits have quite short decoherence times, ~ 1 ns [3,10], limited by the background charge fluctuations, but offer some advantages, e.g., demonstrated simplicity of the qubit-qubit coupling [10]. Therefore it would be of interest to use the approach discussed in this work to suppress dephasing of charge degrees of freedom with the help of controlled flux dynamics. A sketch of the possible set-up achieving this is shown in Fig. 1. Its main elements are the charge qubits, formed by two small tunnel junctions in series, enclosed in small superconducting loops threaded by magnetic flux Φ equal to half of the magnetic flux quantum Φ_0. It can be shown [16] that the current in each loop represents the σ_x component of the qubit dynamics, and its monitoring measures therefore the qubit in the σ_x basis as needed for detection of the dephasing errors. Comparison of the states of the nearest-neighbor qubits can be achieved by measuring not directly the currents in the loops, but the square of the difference (or of the sum) between the currents. Such a quadratic detection measures the product operator $\sigma_x^{(j)}\sigma_x^{(j+1)}$:

$$(\sigma_x^{(j)} \pm \sigma_x^{(j+1)})^2 = 2(1 \pm \sigma_x^{(j)}\sigma_x^{(j+1)}), \qquad (11)$$

and provides information on whether the states of the two qubits are the same or not without measuring them. Quadratic detection can be realized by the usual mag-

netometers but operated at a point where the linear response coefficient vanishes. These measurements, subsequent classical calculations, and application of correction pulses, can be done with sufficient frequency by existing "SFQ" superconductor electronics [24] compatible with the qubits.

In summary, we suggested a simple scheme of performing basic error-correction in Josephson-junction qubits. The scheme suppresses dephasing errors and can be analyzed quantitatively within the realistic model of environment, including the possibility of noise correlations at different qubits. If the errors introduced by the correction procedure are negligible, the residual dephasing rate for the encoded quantum information decreases exponentially with the degree of encoding.

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